CONNECTED MATHEMATICS® 4

Program Overview

MICHIGAN STATE UNIVERSITY

Connected Mathematics®

LAB AIDS®

Unit Titles
by Grade Level

Grade 6

- **Variables and Patterns:** Introducing Algebraic Reasoning
- **Number Connections:** Expressing Factors and Multiples Algebraically
- **Comparing Quantities:** Ratios, Rates, and Equivalence
- **Bits of Rational:** Extending Fraction Operations and Solving Equations
- **Covering and Surrounding**: Two- and Three-Dimensional Measurement
- **Points of Rational:** A Focus on Decimals and Algebraic Reasoning
- **Data About Us:** Statistics and Data Analysis

Grade 7

- **Shapes and Designs:** Generalizing and Using Properties of Geometric Shapes
- **Completely Rational**: A Focus on Integers
- **Stretching and Shrinking:** Proportional Reasoning in the Context of Similarity
- **Comparing and Scaling:** Proportional Reasoning in the Context of Number
- **Moving Straight Ahead:** Linear Relationships, Expressions, and Equations
- **How Likely Is It?** Proportional Reasoning in the Context of Probability
- **Filling and Wrapping:** Two- and Three-Dimensional Measurement
- **Samples and Populations:** Making Comparisons and Predictions

Grade 8 & Algebra*

- **Thinking with Mathematical Models:** Linear Functions and Bivariate Data
- **Looking for Pythagoras:** The Pythagorean Theorem and Real Numbers
- **Growing, Growing, Growing:** Linear Versus Exponential Patterns of Change
- **Mars, Gravity, and Painted Cubes:** Linear Versus Quadratic Patterns of Change
- **Flip, Spin, Slide, and Stretch:** Exploring Transformations
- **Say It with Symbols:** Reasoning with Equivalent Expressions and Equations
- **It's in the System:** Systems of Linear Equations and Inequalities
- **Function Junction:** A Deeper Look at Algebra and Functions

*The Grade 8 units in *Connected Mathematics® 4* integrate Grade 8 and High School Course I standards. This provides an equitable opportunity for each student to pursue Grade 8 and/or High School Course I by using the same set of materials.

Contents

Connected Mathematics® 4 is developed by a team of educators and teachers at Michigan State University. For more than thirty years, the team at Michigan State University has been designing, field-testing, and evaluating four revisions of the Connected Mathematics Project's (CMP) curriculum, Connected Mathematics®. Each edition reflects the information gathered from research and interactions with students, teachers, administrators, teacher educators, and researchers across the United States and several international countries on what it means to understand an important mathematical idea. This knowledge is then used to create an environment where students and teachers come together to define and solve problems with reasoning, insight, inventiveness, joy, and technical proficiency.

Contents

Connected Mathematics®

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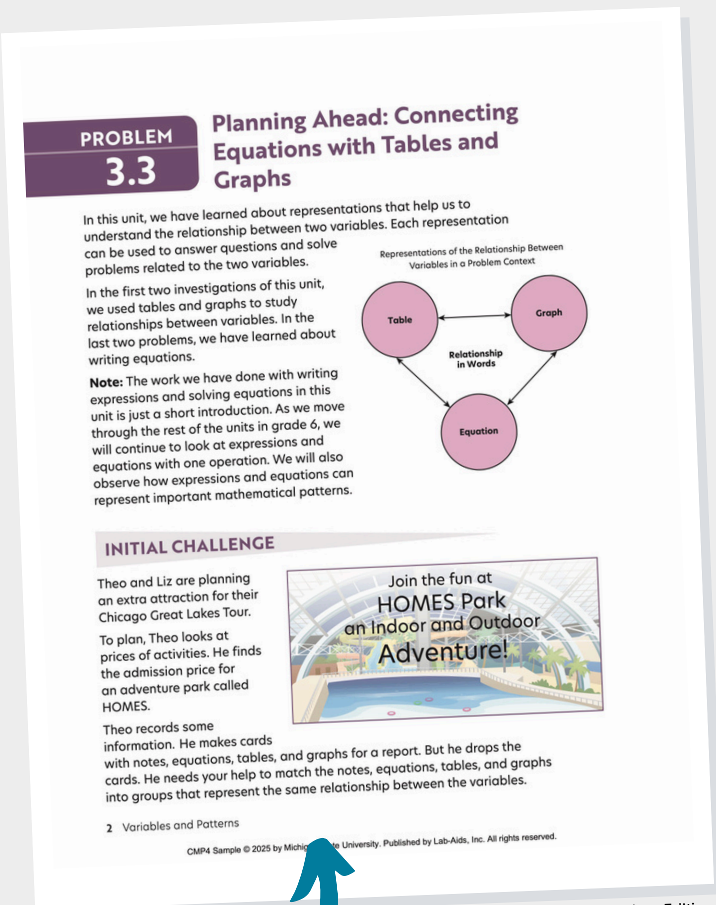
Engage Students
in Deep Connected Mathematics

Connected Mathematics®

The first unit in Grade 6 introduces expressions and equations, setting the foundation for their use in various contexts in grades 6-8, including representing strategies and patterns in algebraic language. This early introduction allows students to apply algebraic language across math strands and solve equations throughout the school year.

Contextualized, Problem-Based Curriculum


PROBLEM 3.3 Planning Ahead: Connecting Equations with Tables and Graphs

In this unit, we have learned about representations that help us to understand the relationship between two variables. Each representation can be used to answer questions and solve problems related to the two variables.

In the first two investigations of this unit, we used tables and graphs to study relationships between variables. In the last two problems, we have learned about writing equations.

Note: The work we have done with writing expressions and solving equations in this unit is just a short introduction. As we move through the rest of the units in grade 6, we will continue to look at expressions and equations with one operation. We will also observe how expressions and equations can represent important mathematical patterns.

INITIAL CHALLENGE

Theo and Liz are planning an extra attraction for their Chicago Great Lakes Tour.

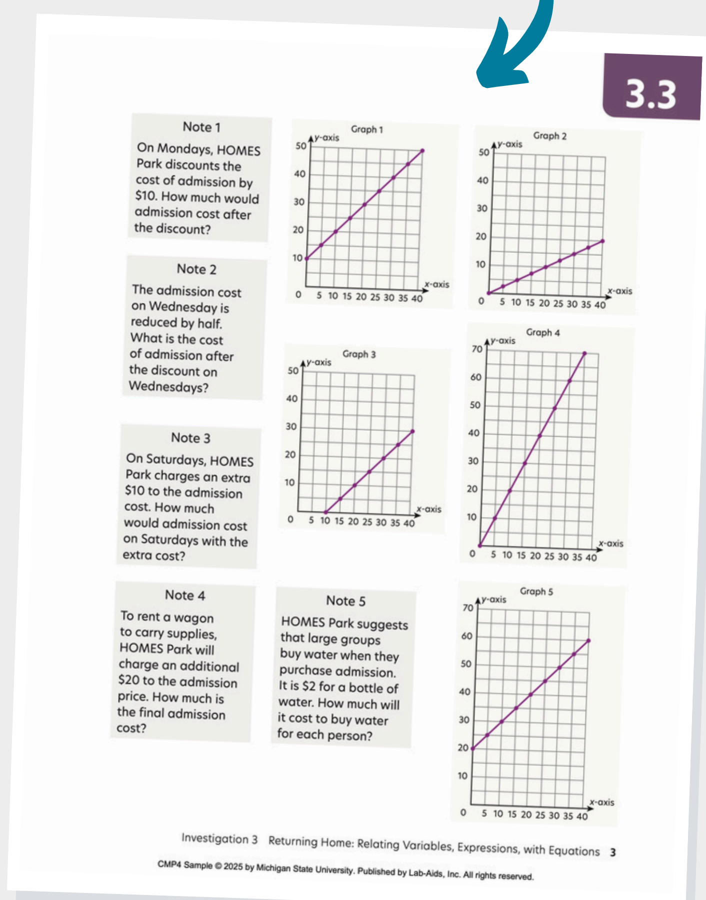
To plan, Theo looks at prices of activities. He finds the admission price for an adventure park called HOMES.

Theo records some information. He makes cards with notes, equations, tables, and graphs for a report. But he drops the cards. He needs your help to match the notes, equations, tables, and graphs into groups that represent the same relationship between the variables.

2 Variables and Patterns

Grade 6 Student Edition

Embedded card sorts, models, matching, games, and experiments are found throughout.



Investigation 3 Returning Home: Relating Variables, Expressions, with Equations 3

Grade 6 Student Edition

Problems are contextualized, providing students with opportunities to make sense of the world and empower them to use mathematics to solve problems.

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Engage Students
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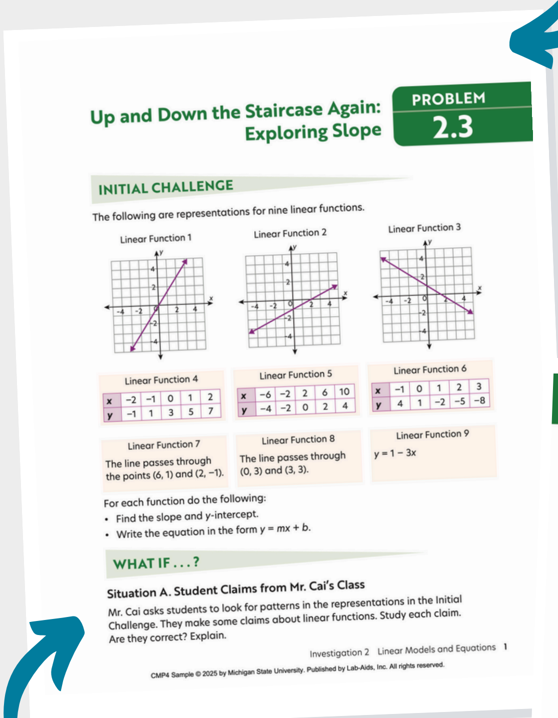
The STEM problem format promotes student engagement and learning as students collaborate to design solutions, make conjectures, offer critiques, and communicate their mathematical understandings. The STEM problem format provides teachers with flexibility to carry out equitable practices that help address the individual needs of all students.

The Initial Challenge poses the mathematical challenge. It provides open access for students.

STEM Problem Format


Up and Down the Staircase Again: Exploring Slope

PROBLEM 2.3

INITIAL CHALLENGE

The following are representations for nine linear functions.

Linear Function 1

Linear Function 2

Linear Function 3

Linear Function 4

x	−2	−1	0	1	2
y	−1	1	3	5	7

Linear Function 5

x	−6	−2	2	6	10
y	−4	−2	0	2	4

Linear Function 6

x	−1	0	1	2	3
y	4	1	−2	−5	−8

Linear Function 7
The line passes through the points (6, 1) and (2, −1).

Linear Function 8
The line passes through (0, 3) and (3, 3).

Linear Function 9
$y = 1 - 3x$

For each function do the following:
- Find the slope and y-intercept.
- Write the equation in the form $y = mx + b$.

WHAT IF . . . ?

Situation A. Student Claims from Mr. Cai's Class

Mr. Cai asks students to look for patterns in the representations in the Initial Challenge. They make some claims about linear functions. Study each claim. Are they correct? Explain.

Investigation 2 Linear Models and Equations 1

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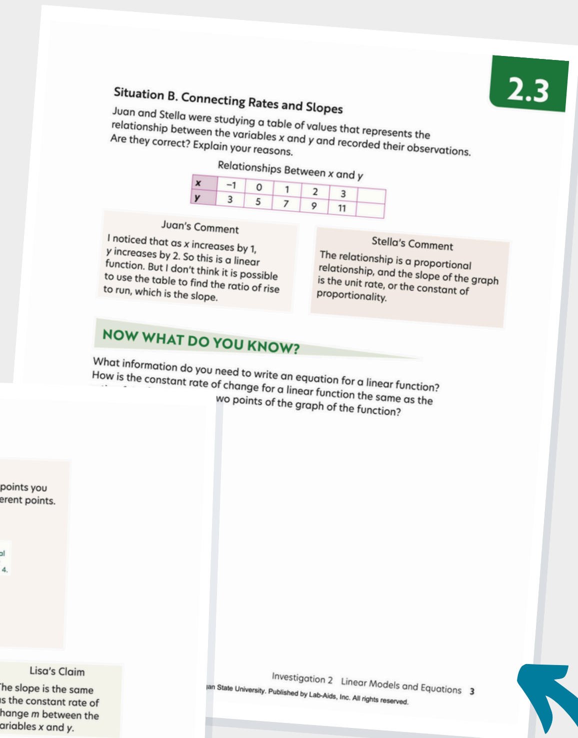
Grade 8 Student Edition


2.3

Situation B. Connecting Rates and Slopes

Juan and Stella were studying a table of values that represents the relationship between the variables x and y and recorded their observations. Are they correct? Explain your reasons.

Relationships Between x and y

x	−1	0	1	2	3	
y	3	5	7	9	11	

Juan's Comment
I noticed that as x increases by 1, y increases by 2. So this is a linear function. But I don't think it is possible to use the table to find the ratio of rise to run, which is the slope.

Stella's Comment
The relationship is a proportional relationship, and the slope of the graph is the unit rate, or the constant of proportionality.

NOW WHAT DO YOU KNOW?

What information do you need to write an equation for a linear function?
How is the constant rate of change for a linear function the same as the
wo points of the graph of the function?

Investigation 2 Linear Models and Equations 3

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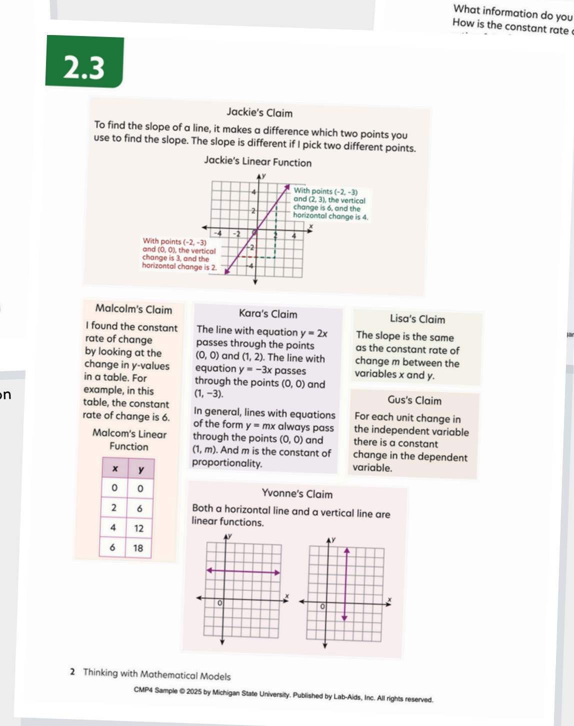
Grade 8 Student Edition


2.3

Jackie's Claim
To find the slope of a line, it makes a difference which two points you use to find the slope. The slope is different if I pick two different points.

Jackie's Linear Function

With points (-2, -3) and (2, 3), the vertical change is 6, and the horizontal change is 4.

With points (-2, -3) and (0, 0), the vertical change is 3, and the horizontal change is 2.

Malcolm's Claim
I found the constant rate of change by looking at the change in y-values in a table. For example, in this table, the constant rate of change is 6.

Malcom's Linear Function

x	y
0	0
2	6
4	12
6	18

Kara's Claim
The line with equation $y = 2x$ passes through the points (0, 0) and (1, 2). The line with equation $y = -3x$ passes through the points (0, 0) and (1, −3).
In general, lines with equations of the form $y = mx$ always pass through the points (0, 0) and (1, m). And m is the constant of proportionality.

Lisa's Claim
The slope is the same as the constant rate of change m between the variables x and y.

Gus's Claim
For each unit change in the independent variable there is a constant change in the dependent variable.

Yvonne's Claim
Both a horizontal line and a vertical line are linear functions.

2 Thinking with Mathematical Models

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Grade 8 Student Edition

The What If...? situations unpack the mathematical understandings. Students consider different situations with different quantities, contexts, or strategies.

The Now What Do You Know? connects the embedded understandings with prior and future knowledge.

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Engage Students
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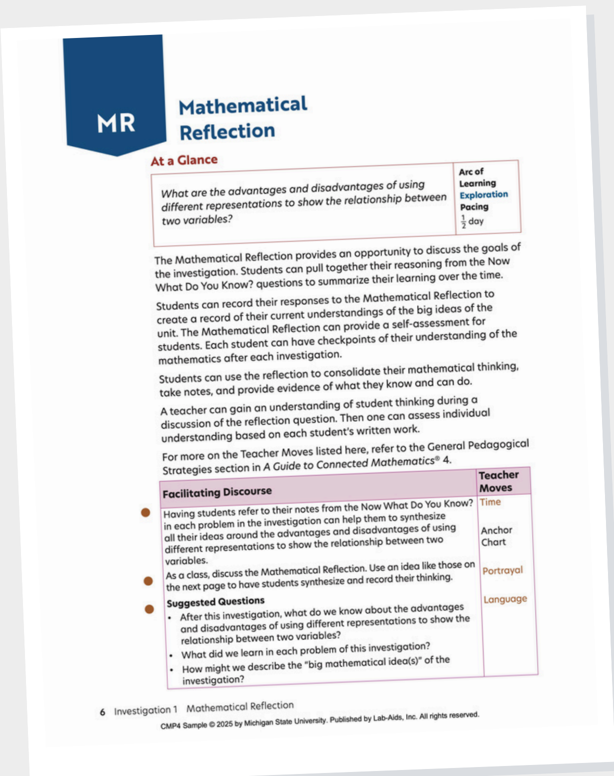
The Mathematical Reflection (MR) for each unit consists of one overarching question that guides the development of the big mathematical idea(s) in the unit. By receiving feedback from teachers and using other students' work to provoke new ideas, students can learn to reflect on their own progress in making sense of mathematics.

Mathematical Reflection

The MR provides an opportunity for students to consolidate their learning of the big ideas at a key stage of their development.



MR

Mathematical Reflection

At a Glance

What are the advantages and disadvantages of using different representations to show the relationship between two variables?	**Arc of Learning** Exploration **Pacing** $\frac{1}{2}$ day

The Mathematical Reflection provides an opportunity to discuss the goals of the investigation. Students can pull together their reasoning from the Now What Do You Know? questions to summarize their learning over the time.

Students can record their responses to the Mathematical Reflection to create a record of their current understandings of the big ideas of the unit. The Mathematical Reflection can provide a self-assessment for students. Each student can have checkpoints of their understanding of the mathematics after each investigation.

Students can use the reflection to consolidate their mathematical thinking, take notes, and provide evidence of what they know and can do.

A teacher can gain an understanding of student thinking during a discussion of the reflection question. Then one can assess individual understanding based on each student's written work.

For more on the Teacher Moves listed here, refer to the General Pedagogical Strategies section in *A Guide to Connected Mathematics*® 4.

Facilitating Discourse	Teacher Moves
Having students refer to their notes from the Now What Do You Know? in each problem in the investigation can help them to synthesize all their ideas around the advantages and disadvantages of using different representations to show the relationship between two variables.	Time Anchor Chart
As a class, discuss the Mathematical Reflection. Use an idea like those on the next page to have students synthesize and record their thinking.	Portrayal
Suggested Questions • After this investigation, what do we know about the advantages and disadvantages of using different representations to show the relationship between two variables? • What did we learn in each problem of this investigation? • How might we describe the "big mathematical idea(s)" of the investigation?	Language

6 Investigation 1 Mathematical Reflection

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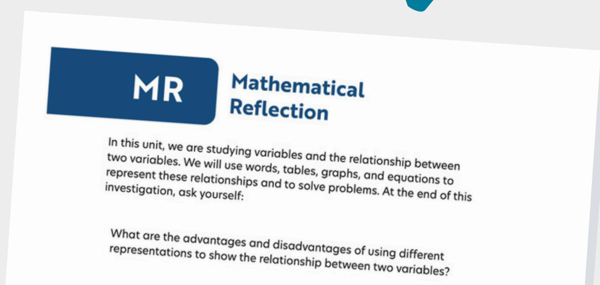
Grade 6 Teacher Edition



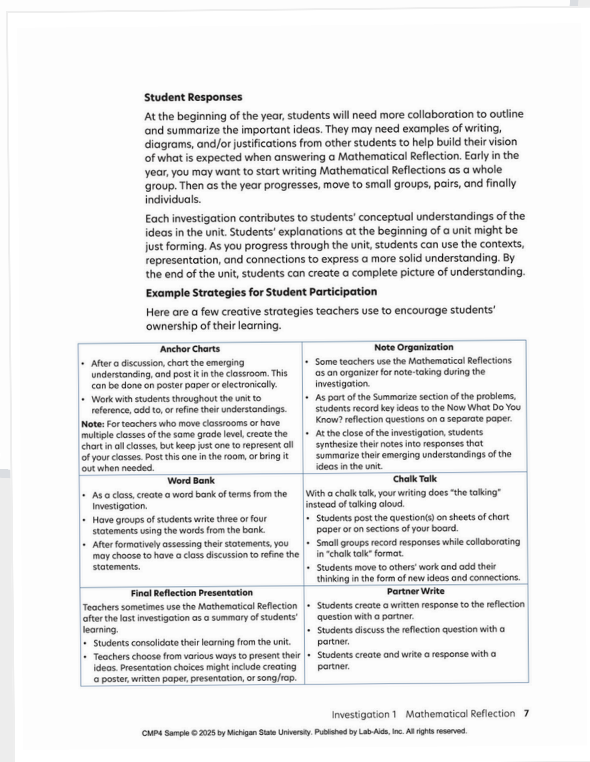
MR

Mathematical Reflection

In this unit, we are studying variables and the relationship between two variables. We will use words, tables, graphs, and equations to represent these relationships and to solve problems. At the end of this investigation, ask yourself:

What are the advantages and disadvantages of using different representations to show the relationship between two variables?

Grade 6 Student Edition



Student Responses

At the beginning of the year, students will need more collaboration to outline and summarize the important ideas. They may need examples of writing, diagrams, and/or justifications from other students to help build their vision of what is expected when answering a Mathematical Reflection. Early in the year, you may want to start writing Mathematical Reflections as a whole group. Then as the year progresses, move to small groups, pairs, and finally individuals.

Each investigation contributes to students' conceptual understandings of the ideas in the unit. Students' explanations at the beginning of a unit might be just forming. As you progress through the unit, students can use the contexts, representation, and connections to express a more solid understanding. By the end of the unit, students can create a complete picture of understanding.

Example Strategies for Student Participation

Here are a few creative strategies teachers use to encourage students' ownership of their learning.

Anchor Charts	Note Organization
• After a discussion, chart the emerging understanding, and post it in the classroom. This can be done on poster paper or electronically. • Work with students throughout the unit to reference, add to, or refine their understandings. **Note:** For teachers who move classrooms or have multiple classes of the same grade level, create the chart in all classes, but keep just one to represent all of your classes. Post this one in the room, or bring it out when needed.	• Some teachers use the Mathematical Reflections as an organizer for note-taking during the investigation. • As part of the Summarize section of the problems, students record key ideas to the Now What Do You Know? reflection questions on a separate paper. • At the close of the investigation, students synthesize their notes into responses that summarize their emerging understandings of the ideas in the unit.
Word Bank	**Chalk Talk**
• As a class, create a word bank of terms from the Investigation. • Have groups of students write three or four statements using the words from the bank. • After formatively assessing their statements, you may choose to have a class discussion to refine the statements.	With a chalk talk, your writing does "the talking" instead of talking aloud. • Students post the question(s) on sheets of chart paper or on sections of your board. • Small groups record responses while collaborating in "chalk talk" format. • Students move to others' work and add their thinking in the form of new ideas and connections.
Final Reflection Presentation	**Partner Write**
Teachers sometimes use the Mathematical Reflection after the last investigation as a summary of students' learning. • Students consolidate their learning from the unit. • Teachers choose from various ways to present their ideas. Presentation choices might include creating a poster, written paper, presentation, or song/rap.	• Students create a written response to the reflection question with a partner. • Students discuss the reflection question with a partner. • Students create and write a response with a partner.

Investigation 1 Mathematical Reflection 7

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Grade 6 Teacher Edition

For many students, self-assessment is a new experience, and they may initially need more guidance as provided in the Teacher Edition.

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Teacher Resources

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The Arc of Learning™ (AoL) makes the curriculum's intentions explicit about how students progress from informal to more formal mathematical understandings in a unit and across units.

Arc of Learning™

The number of problems in CMP4 is about 20% less than the previous edition, creating tighter units of instruction so teachers can move through the content in a school year.

UNIT ARC OF LEARNING™ (AoL)

Thinking with Mathematical Models: Linear Functions and Bivariate Data (AoL) ■ Linear Functions ■ Bivariate Data	Introduction *Setting the Scene*	Exploration *Mucking About*	Analysis *Going Deeper*	Synthesis *Looking Across*	Abstraction *Going Beyond*
Investigation 1. Exploring Data Patterns in Building Bridges					
Problem 1.1 Bridge Thickness and Strength: Linear or Nonlinear?	1.1	1.1	1.1		
Problem 1.2 Bridge Length and Strength: Linear or Nonlinear?	1.2	1.2	1.2		
Problem 1.3 Custom Construction Parts: More Patterns	1.3	1.3	1.3		
Mathematical Reflection	MR		MR		
Investigation 2. Linear Models and Equations					
Problem 2.1 Treetop Fun: Equations for Linear Functions	2.1		2.1	2.1	
Problem 2.2 Boat Rentals: Finding Solutions Using Tables, Graphs, and Symbols	2.2		2.2	2.2	
Problem 2.3 Up and Down the Staircase Again: Exploring Slope	2.3		2.3	2.3	
Problem 2.4 Exploring Patterns with Lines	2.4		2.4	2.4	
Mathematical Reflection	MR		2.5	MR	
Investigation 3. Inverse Variation: Linear or Nonlinear?					
Problem 3.1 Rectangles with Fixed Area	3.1	3.1	3.1	3.1	
Problem 3.2 Distance, Speed, and Time	3.2	3.2		3.2	
Problem 3.3 Planning a Fundraising Event: Finding Individual Costs	3.3	3.3		3.3	
Mathematical Reflection	MR	MR		MR	
Investigation 4. Variability in Numerical and Categorical Data					
Problem 4.1 Lines of Best Fit		4.1	4.1	4.1	
Problem 4.2 Wood or Steel? Relationships in Categorical Data	4.2	4.2			
Problem 4.3 School Team-Building Activity: Setting Up a Two-Way Table	4.3	4.3		4.3	
Problem 4.4 Linear, Bivariate, and Categorical Data		4.4	4.4	4.4	4.4
Mathematical Reflection			MR	MR	MR

Grade 8 Teacher Edition

As students explore a series of connected problems, they develop an understanding of the embedded ideas and, with the aid of the teacher, abstract powerful mathematical ideas, problem-solving strategies, and ways of thinking.

Linear relationships were used in *Moving Straight Ahead* to provide contextual settings for studying linear expressions and equations. So students come to *Thinking with Mathematical Models* with a solid basis to build on and expand their understanding of linear functions. This is done by looking at both linear and nonlinear functions. The nonlinear function in this unit is inverse variation. For example, the relationship distance = rate • time, or $d = rt$, has three variables, d, r, and t. Depending on which variable is a constant, the resulting relationship is either linear or inverse. Students further deepen their understanding of linear functions by exploring scatterplots and deciding which plots have a strong association and can be modeled with a linear function. Bivariate and categorical data are informally introduced and explored. Deeper understanding of these ideas occurs in high school.

Grade 8 Teacher Edition

The AoL characterizes deeply grounded, connected learning, contrasting the common approach of passive, isolated skill acquisition.

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Teacher Resources
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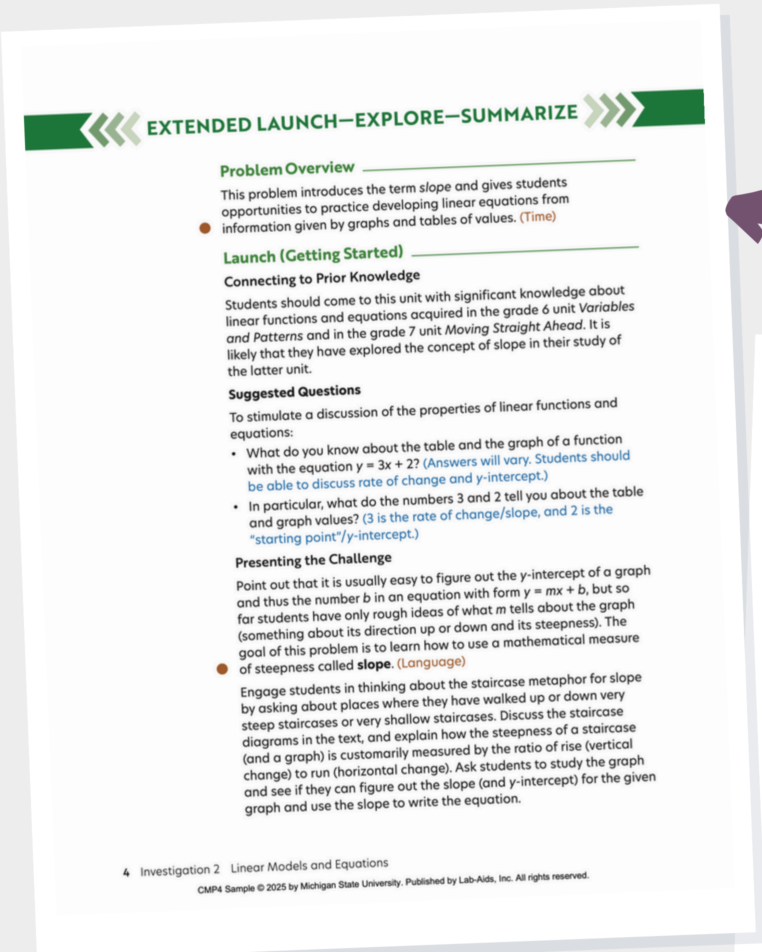
The Teacher Edition materials are organized around an instructional model that supports a problem-solving STEM classroom. The three instructional phases of a lesson are Launch—Explore—Summarize.

Extended Launch—Explore—Summarize Model



EXTENDED LAUNCH—EXPLORE—SUMMARIZE

Problem Overview

This problem introduces the term *slope* and gives students opportunities to practice developing linear equations from information given by graphs and tables of values. (Time)

Launch (Getting Started)

Connecting to Prior Knowledge

Students should come to this unit with significant knowledge about linear functions and equations acquired in the grade 6 unit *Variables and Patterns* and in the grade 7 unit *Moving Straight Ahead*. It is likely that they have explored the concept of slope in their study of the latter unit.

Suggested Questions

To stimulate a discussion of the properties of linear functions and equations:

- What do you know about the table and the graph of a function with the equation $y = 3x + 2$? (Answers will vary. Students should be able to discuss rate of change and y-intercept.)
- In particular, what do the numbers 3 and 2 tell you about the table and graph values? (3 is the rate of change/slope, and 2 is the "starting point"/y-intercept.)

Presenting the Challenge

Point out that it is usually easy to figure out the y-intercept of a graph and thus the number b in an equation with form $y = mx + b$, but so far students have only rough ideas of what m tells about the graph (something about its direction up or down and its steepness). The goal of this problem is to learn how to use a mathematical measure of steepness called **slope**. (Language)

Engage students in thinking about the staircase metaphor for slope by asking about places where they have walked up or down very steep staircases or very shallow staircases. Discuss the staircase diagrams in the text, and explain how the steepness of a staircase (and a graph) is customarily measured by the ratio of rise (vertical change) to run (horizontal change). Ask students to study the graph and see if they can figure out the slope (and y-intercept) for the given graph and use the slope to write the equation.

4 Investigation 2 Linear Models and Equations

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Grade 8 Teacher Edition

Support for the teacher helps them position the new problem within prior understanding and problems and present the challenge to students.

LES

Have students work in partners on the problem. For the Initial Challenge, you may want to assign partners 2 or 3 of the 8 functions. Make sure that each group has positive and negative slope functions. **Learning Aid 2.3: Slope and y-Intercept** can be used so that students can draw on the graphs or tables as needed. (Problem-Solving Environment)

Explore (Digging In)

Providing for Individual Needs

Suggested Questions

- How do you identify the slope and the y-intercept of a line from its graph? (The y-intercept is where the line crosses the y-axis. One way that slope can be determined is to choose two points and find the rise and run between them. As an example, for Linear Function 1: Choosing (0, 0) and (2, 3), the rise would be 3, and the run would be 2. The slope for this example would be $\frac{3}{2}$, or 1.5.)
- How do you identify the slope and the y-intercept of a line from a table? (To find the slope, you find the difference in the y-values between two consecutive values [rise] and then the difference between the corresponding x-values [run]. When $x = 0$, the associated y-value is the y-intercept. As an example, for Linear Function 4:

x	−2	−1	0	1	2
y	−1	1	3	5	7

The difference between −1 and 1 is 2 [rise]. The difference between −2 and −1 is 1 [run]. The slope for this example would be 2, and the y-intercept would be (0, 3).)
- What information do you need to identify the slope of a line and/or other points on a line? (You need the coordinates of two points on the line so that you can find the rise and the run.)
- For Linear Function 7, how do you find the information you need to write an equation? (Answers will vary. Students might make a table and figure out the constant rate is $\frac{1}{2}$ and then fill out the table to figure out the y-intercept. Or they might connect the two points on a graph and try to read the intercept. Or they might figure the slope from the two given points, write $y = 0.5x + b$, and substitute a data pair to find b.)
- How do you use slope and y-intercept to find the equation of a line? (To write the equation of a line, substitute the value of slope for m and the value of the y-intercept for b in the standard equation $y = mx + b$.)

Extended Launch—Explore—Summarize 5

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Grade 8 Teacher Edition

Suggested questions and pedagogical strategies are provided for teachers as they observe and interact with individual and small groups working on the problem.

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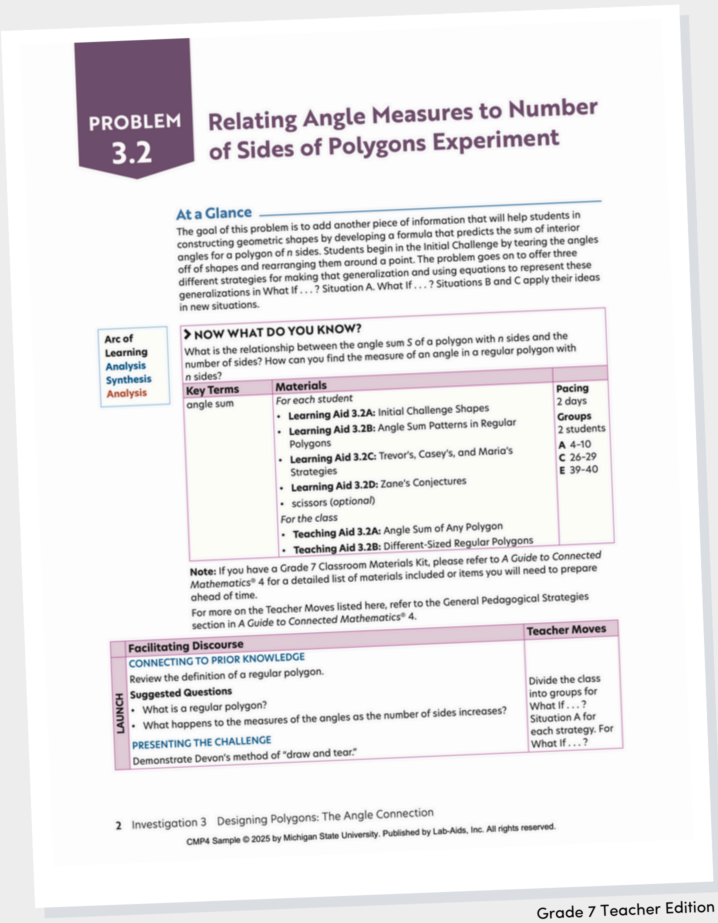
The At a Glance section provides an overview of the problem that includes a description of the problem context, the Arc of Learning™ phase(s), and the Now What Do You Know? This section highlights key pieces found within the Extended Launch-Explore-Summarize teacher support.

At a Glance

Provides planning information that includes materials needed to implement the problem, recommended ACE, and suggestions for student grouping.



PROBLEM 3.2

Relating Angle Measures to Number of Sides of Polygons Experiment

At a Glance

The goal of this problem is to add another piece of information that will help students in constructing geometric shapes by developing a formula that predicts the sum of interior angles for a polygon of *n* sides. Students begin in the Initial Challenge by tearing the angles off of shapes and rearranging them around a point. The problem goes on to offer three different strategies for making that generalization and using equations to represent these generalizations in What If . . . ? Situation A. What If . . . ? Situations B and C apply their ideas in new situations.

Arc of Learning
Analysis
Synthesis
Analysis

> NOW WHAT DO YOU KNOW?
What is the relationship between the angle sum *S* of a polygon with *n* sides and the number of sides? How can you find the measure of an angle in a regular polygon with *n* sides?

Key Terms	Materials	
angle sum	*For each student* • **Learning Aid 3.2A:** Initial Challenge Shapes • **Learning Aid 3.2B:** Angle Sum Patterns in Regular Polygons • **Learning Aid 3.2C:** Trevor's, Casey's, and Maria's Strategies • **Learning Aid 3.2D:** Zane's Conjectures • scissors (*optional*) *For the class* • **Teaching Aid 3.2A:** Angle Sum of Any Polygon • **Teaching Aid 3.2B:** Different-Sized Regular Polygons	**Pacing** 2 days **Groups** 2 students **A** 4–10 **C** 26–29 **E** 39–40

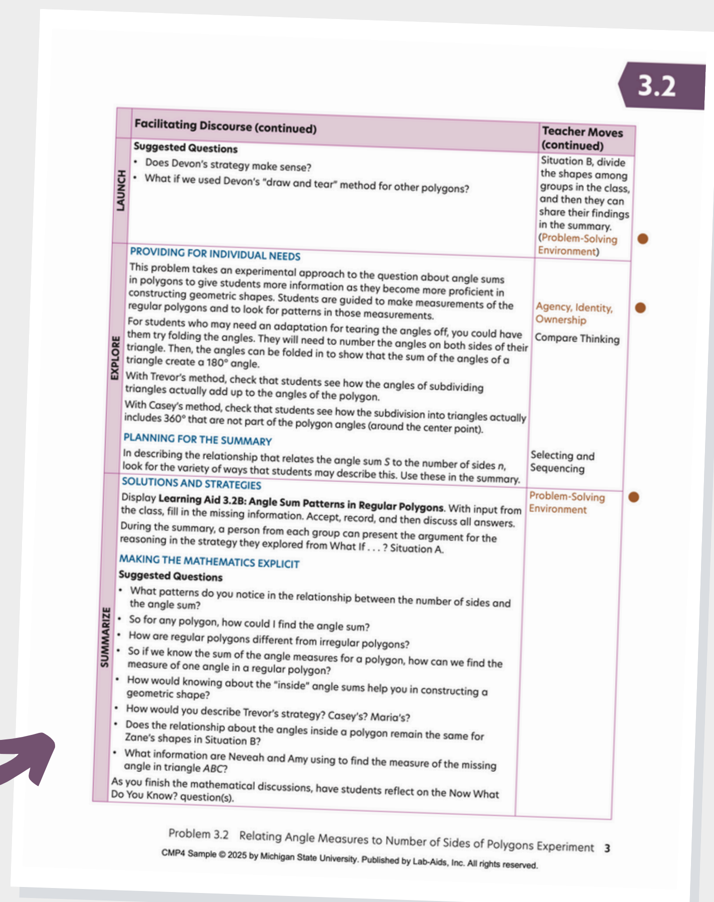
Note: If you have a Grade 7 Classroom Materials Kit, please refer to *A Guide to Connected Mathematics® 4* for a detailed list of materials included or items you will need to prepare ahead of time.

For more on the Teacher Moves listed here, refer to the General Pedagogical Strategies section in *A Guide to Connected Mathematics® 4*.

	Facilitating Discourse	Teacher Moves
LAUNCH	CONNECTING TO PRIOR KNOWLEDGE Review the definition of a regular polygon. **Suggested Questions** • What is a regular polygon? • What happens to the measures of the angles as the number of sides increases? PRESENTING THE CHALLENGE Demonstrate Devon's method of "draw and tear."	Divide the class into groups for What If . . . ? Situation A for each strategy. For What If . . . ?

2 Investigation 3 Designing Polygons: The Angle Connection

Grade 7 Teacher Edition



3.2

	Facilitating Discourse (continued)	Teacher Moves (continued)
LAUNCH	**Suggested Questions** • Does Devon's strategy make sense? • What if we used Devon's "draw and tear" method for other polygons?	Situation B, divide the shapes among groups in the class, and then they can share their findings in the summary. (Problem-Solving Environment)
EXPLORE	PROVIDING FOR INDIVIDUAL NEEDS This problem takes an experimental approach to the question about angle sums in polygons to give students more information as they become more proficient in constructing geometric shapes. Students are guided to make measurements of the regular polygons and to look for patterns in those measurements. For students who may need an adaptation for tearing the angles off, you could have them try folding the angles. They will need to number the angles on both sides of their triangle. Then, the angles can be folded in to show that the sum of the angles of a triangle create a 180° angle. With Trevor's method, check that students see how the angles of subdividing triangles actually add up to the angles of the polygon. With Casey's method, check that students see how the subdivision into triangles actually includes 360° that are not part of the polygon angles (around the center point). PLANNING FOR THE SUMMARY In describing the relationship that relates the angle sum *S* to the number of sides *n*, look for the variety of ways that students may describe this. Use these in the summary.	Agency, Identity, Ownership Compare Thinking Selecting and Sequencing
SUMMARIZE	SOLUTIONS AND STRATEGIES Display **Learning Aid 3.2B: Angle Sum Patterns in Regular Polygons**. With input from the class, fill in the missing information. Accept, record, and then discuss all answers. During the summary, a person from each group can present the argument for the reasoning in the strategy they explored from What If . . . ? Situation A. MAKING THE MATHEMATICS EXPLICIT **Suggested Questions** • What patterns do you notice in the relationship between the number of sides and the angle sum? • So for any polygon, how could I find the angle sum? • How are regular polygons different from irregular polygons? • So if we know the sum of the angle measures for a polygon, how can we find the measure of one angle in a regular polygon? • How would knowing about the "inside" angle sums help you in constructing a geometric shape? • How would you describe Trevor's strategy? Casey's? Maria's? • Does the relationship about the angles inside a polygon remain the same for Zane's shapes in Situation B? • What information are Neveah and Amy using to find the measure of the missing angle in triangle *ABC*? As you finish the mathematical discussions, have students reflect on the Now What Do You Know? question(s).	Problem-Solving Environment

Problem 3.2 Relating Angle Measures to Number of Sides of Polygons Experiment 3

Grade 7 Teacher Edition

Includes suggested questions for each phase of the lesson, suggested teacher moves, and suggestions for connections to the Attending to Individual Learning Needs Framework.

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LAB AIDS®

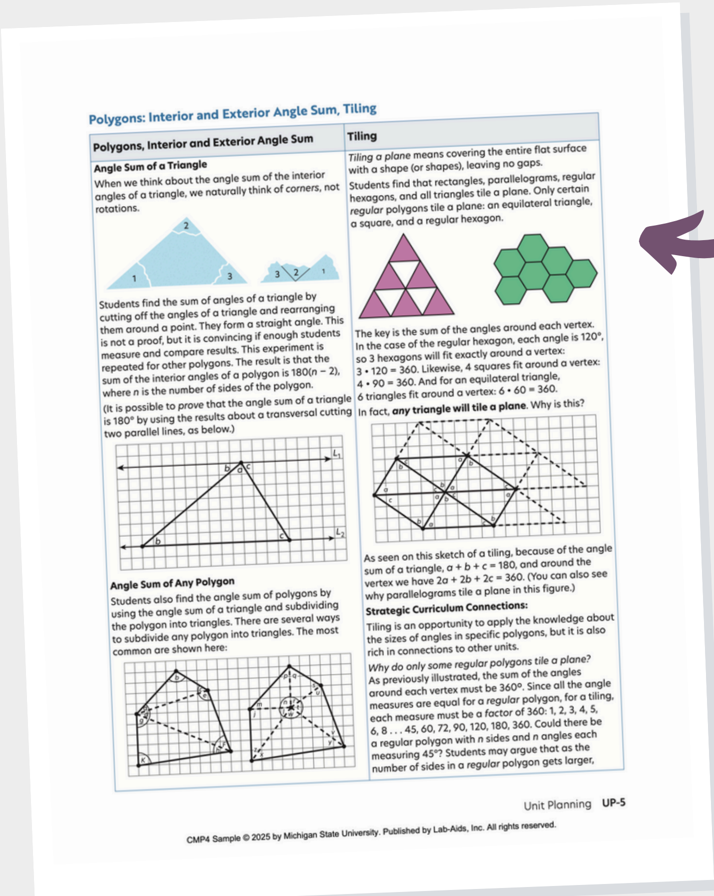
Teacher Resources

and Professional Support

Connected Mathematics®

The Mathematics Overview supports teachers' understanding of the mathematical development in a unit and the rich connections among mathematical strands in prior and future units.

Mathematics Overview



Polygons: Interior and Exterior Angle Sum, Tiling

Polygons, Interior and Exterior Angle Sum

Angle Sum of a Triangle

When we think about the angle sum of the interior angles of a triangle, we naturally think of *corners*, not rotations.

Students find the sum of angles of a triangle by cutting off the angles of a triangle and rearranging them around a point. They form a straight angle. This is not a proof, but it is convincing if enough students measure and compare results. This experiment is repeated for other polygons. The result is that the sum of the interior angles of a polygon is $180(n - 2)$, where n is the number of sides of the polygon.

(It is possible to *prove* that the angle sum of a triangle is 180° by using the results about a transversal cutting two parallel lines, as below.)

Angle Sum of Any Polygon

Students also find the angle sum of polygons by using the angle sum of a triangle and subdividing the polygon into triangles. There are several ways to subdivide any polygon into triangles. The most common are shown here:

Tiling

Tiling a plane means covering the entire flat surface with a shape (or shapes), leaving no gaps.

Students find that rectangles, parallelograms, regular hexagons, and all triangles tile a plane. Only certain *regular* polygons tile a plane: an equilateral triangle, a square, and a regular hexagon.

The key is the sum of the angles around each vertex. In the case of the regular hexagon, each angle is 120°, so 3 hexagons will fit exactly around a vertex: $3 \cdot 120 = 360$. Likewise, 4 squares fit around a vertex: $4 \cdot 90 = 360$. And for an equilateral triangle, 6 triangles fit around a vertex: $6 \cdot 60 = 360$.

In fact, **any triangle will tile a plane**. Why is this?

As seen on this sketch of a tiling, because of the angle sum of a triangle, $a + b + c = 180$, and around the vertex we have $2a + 2b + 2c = 360$. (You can also see why parallelograms tile a plane in this figure.)

Strategic Curriculum Connections:

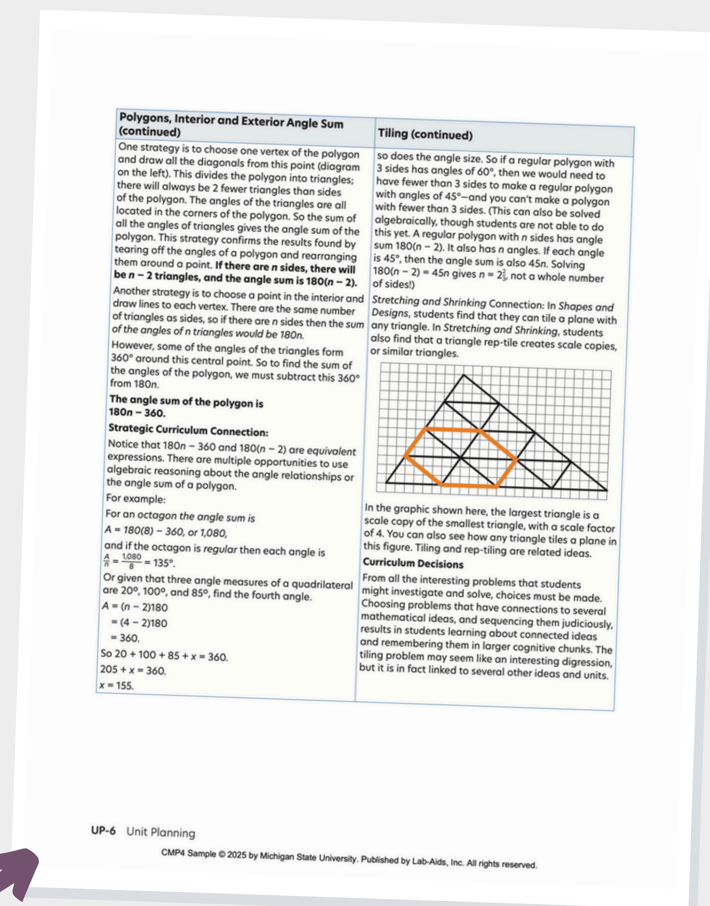
Tiling is an opportunity to apply the knowledge about the sizes of angles in specific polygons, but it is also rich in connections to other units.

Why do only some regular polygons tile a plane? As previously illustrated, the sum of the angles around each vertex must be 360°. Since all the angle measures are equal for a *regular* polygon, for a tiling, each measure must be a *factor* of 360: 1, 2, 3, 4, 5, 6, 8 . . . 45, 60, 72, 90, 120, 180, 360. Could there be a regular polygon with n sides and n angles each measuring 45°? Students may argue that as the number of sides in a *regular* polygon gets larger,

Unit Planning **UP-5**

Grade 7 Teacher Edition

Strategic Curriculum Connections describe how mathematical concepts and ideas developed in a unit connect to prior and/or future learning.



Polygons, Interior and Exterior Angle Sum (continued)

One strategy is to choose one vertex of the polygon and draw all the diagonals from this point (diagram on the left). This divides the polygon into triangles; there will always be 2 fewer triangles than sides of the polygon. The angles of the triangles are all located in the corners of the polygon. So the sum of all the angles of triangles gives the angle sum of the polygon. This strategy confirms the results found by tearing off the angles of a polygon and rearranging them around a point. **If there are n sides, there will be $n - 2$ triangles, and the angle sum is $180(n - 2)$.**

Another strategy is to choose a point in the interior and draw lines to each vertex. There are the same number of triangles as sides, so if there are n sides then the *sum of the angles of n triangles would be 180n.*

However, some of the angles of the triangles form 360° around this central point. So to find the sum of the angles of the polygon, we must subtract this 360° from $180n$.

The angle sum of the polygon is $180n - 360$.

Strategic Curriculum Connection:

Notice that $180n - 360$ and $180(n - 2)$ are *equivalent* expressions. There are multiple opportunities to use algebraic reasoning about the angle relationships or the angle sum of a polygon.

For example:

For an octagon the angle sum is

$A = 180(8) - 360$, or 1,080,

and if the octagon is *regular* then each angle is $\frac{A}{n} = \frac{1,080}{8} = 135°$.

Or given that three angle measures of a quadrilateral are 20°, 100°, and 85°, find the fourth angle.

$A = (n - 2)180$

$= (4 - 2)180$

$= 360.$

So $20 + 100 + 85 + x = 360$.

$205 + x = 360$.

$x = 155$.

Tiling (continued)

so does the angle size. So if a regular polygon with 3 sides has angles of 60°, then we would need to have fewer than 3 sides to make a regular polygon with angles of 45°—and you can't make a polygon with fewer than 3 sides. (This can also be solved algebraically, though students are not able to do this yet. A regular polygon with n sides has angle sum $180(n - 2)$. It also has n angles. If each angle is 45°, then the angle sum is also $45n$. Solving $180(n - 2) = 45n$ gives $n = 2\frac{2}{3}$, not a whole number of sides!)

Stretching and Shrinking Connection: In *Shapes and Designs*, students find that they can tile a plane with any triangle. In *Stretching and Shrinking*, students also find that a triangle rep-tile creates scale copies, or similar triangles.

In the graphic shown here, the largest triangle is a scale copy of the smallest triangle, with a scale factor of 4. You can also see how any triangle tiles a plane in this figure. Tiling and rep-tiling are related ideas.

Curriculum Decisions

From all the interesting problems that students might investigate and solve, choices must be made. Choosing problems that have connections to several mathematical ideas, and sequencing them judiciously, results in students learning about connected ideas and remembering them in larger cognitive chunks. The tiling problem may seem like an interesting digression, but it is in fact linked to several other ideas and units.

UP-6 Unit Planning

Grade 7 Teacher Edition

New concepts and skills connect to and build on prior understandings. As a result, students deepen their understandings or prior learnings at the same time they are developing the new understandings.

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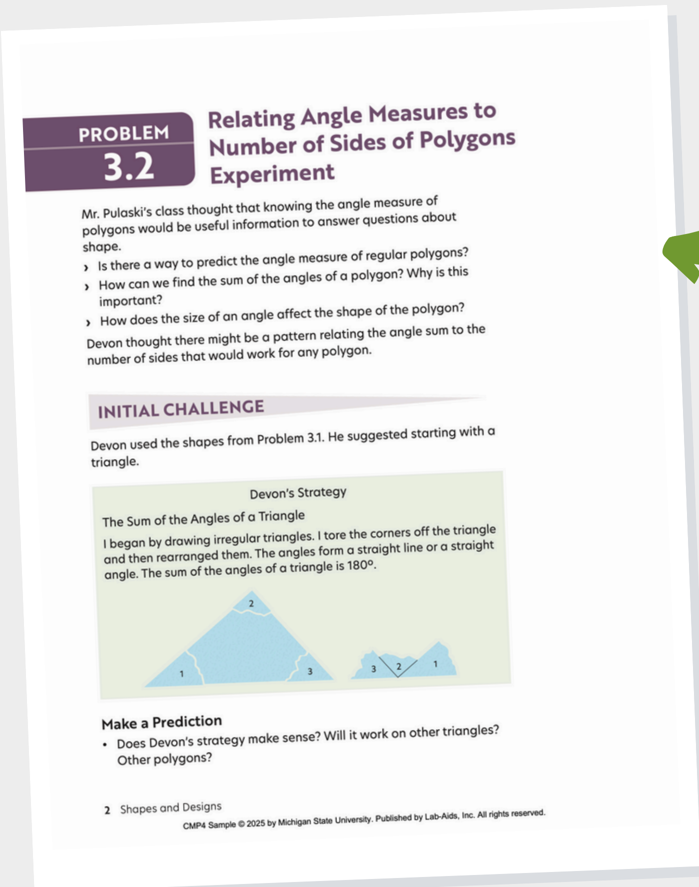
LAB AIDS®

Equity
and Differentiation

Connected Mathematics

Connected Mathematics® 4 holds high expectations for each and every student. The STEM problem format provides teachers with flexibility to carry out equitable practices that help address the individual needs of all students.

STEM Problem Format



PROBLEM 3.2

Relating Angle Measures to Number of Sides of Polygons Experiment

Mr. Pulaski's class thought that knowing the angle measure of polygons would be useful information to answer questions about shape.

- Is there a way to predict the angle measure of regular polygons?
- How can we find the sum of the angles of a polygon? Why is this important?
- How does the size of an angle affect the shape of the polygon?

Devon thought there might be a pattern relating the angle sum to the number of sides that would work for any polygon.

INITIAL CHALLENGE

Devon used the shapes from Problem 3.1. He suggested starting with a triangle.

Devon's Strategy

The Sum of the Angles of a Triangle

I began by drawing irregular triangles. I tore the corners off the triangle and then rearranged them. The angles form a straight line or a straight angle. The sum of the angles of a triangle is 180°.

Make a Prediction

- Does Devon's strategy make sense? Will it work on other triangles? Other polygons?

2 Shapes and Designs

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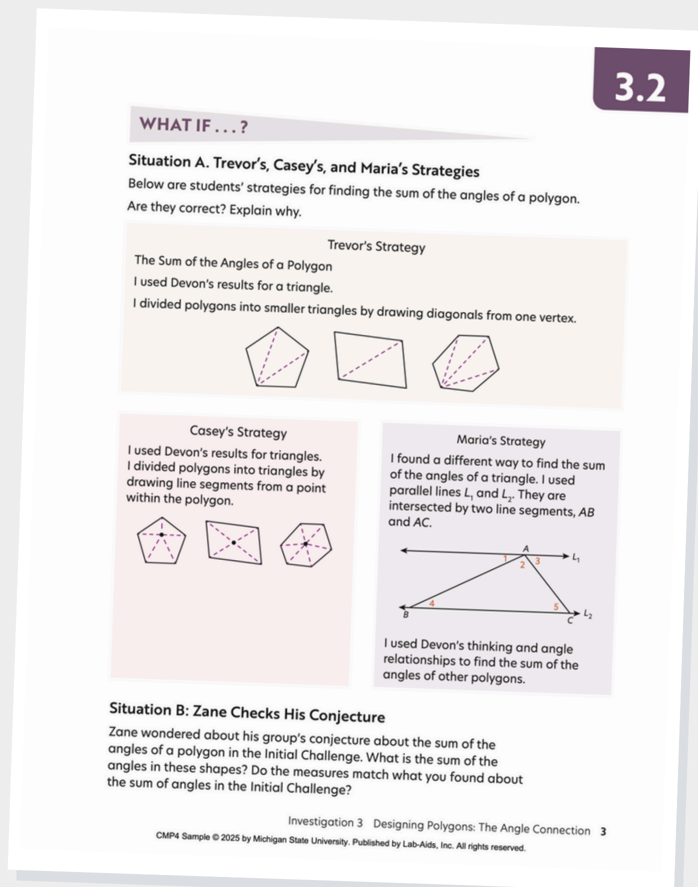
Grade 7 Student Edition

Challenging problems provide students with multiple entry points and differentiation throughout the lesson.



3.2

WHAT IF . . . ?

Situation A. Trevor's, Casey's, and Maria's Strategies

Below are students' strategies for finding the sum of the angles of a polygon. Are they correct? Explain why.

Trevor's Strategy

The Sum of the Angles of a Polygon

I used Devon's results for a triangle.

I divided polygons into smaller triangles by drawing diagonals from one vertex.

Casey's Strategy

I used Devon's results for triangles. I divided polygons into triangles by drawing line segments from a point within the polygon.

Maria's Strategy

I found a different way to find the sum of the angles of a triangle. I used parallel lines L_1 and L_2. They are intersected by two line segments, *AB* and *AC*.

I used Devon's thinking and angle relationships to find the sum of the angles of other polygons.

Situation B: Zane Checks His Conjecture

Zane wondered about his group's conjecture about the sum of the angles of a polygon in the Initial Challenge. What is the sum of the angles in these shapes? Do the measures match what you found about the sum of angles in the Initial Challenge?

Investigation 3 Designing Polygons: The Angle Connection 3

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Grade 7 Student Edition

Teachers can choose which student groups work on different What If...? situations based on their current understandings.

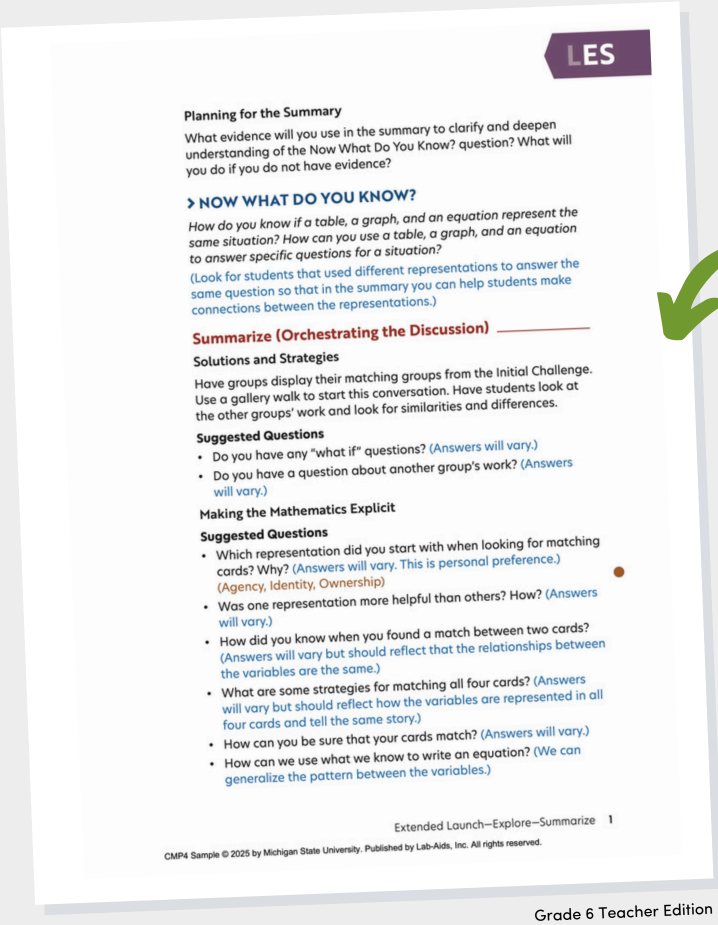
Stay up -to-date with the latest information for early adopters.

LAB AIDS®

The Attending to Individual Learning Needs (AILN) framework characterizes five essential classroom elements for creating an environment in which teachers can support students' development of mathematical identities.

Attending to Individual Learning Needs

Specific suggestions that align with the AILN framework are provided in the Extended Launch-Explore-Summarize to support teachers in addressing the learning needs of all students.



LES

Planning for the Summary

What evidence will you use in the summary to clarify and deepen understanding of the Now What Do You Know? question? What will you do if you do not have evidence?

> NOW WHAT DO YOU KNOW?

How do you know if a table, a graph, and an equation represent the same situation? How can you use a table, a graph, and an equation to answer specific questions for a situation?

(Look for students that used different representations to answer the same question so that in the summary you can help students make connections between the representations.)

Summarize (Orchestrating the Discussion)

Solutions and Strategies

Have groups display their matching groups from the Initial Challenge. Use a gallery walk to start this conversation. Have students look at the other groups' work and look for similarities and differences.

Suggested Questions

- Do you have any "what if" questions? (Answers will vary.)
- Do you have a question about another group's work? (Answers will vary.)

Making the Mathematics Explicit

Suggested Questions

- Which representation did you start with when looking for matching cards? Why? (Answers will vary. This is personal preference.) (Agency, Identity, Ownership)
- Was one representation more helpful than others? How? (Answers will vary.)
- How did you know when you found a match between two cards? (Answers will vary but should reflect that the relationships between the variables are the same.)
- What are some strategies for matching all four cards? (Answers will vary but should reflect how the variables are represented in all four cards and tell the same story.)
- How can you be sure that your cards match? (Answers will vary.)
- How can we use what we know to write an equation? (We can generalize the pattern between the variables.)

Extended Launch—Explore—Summarize 1

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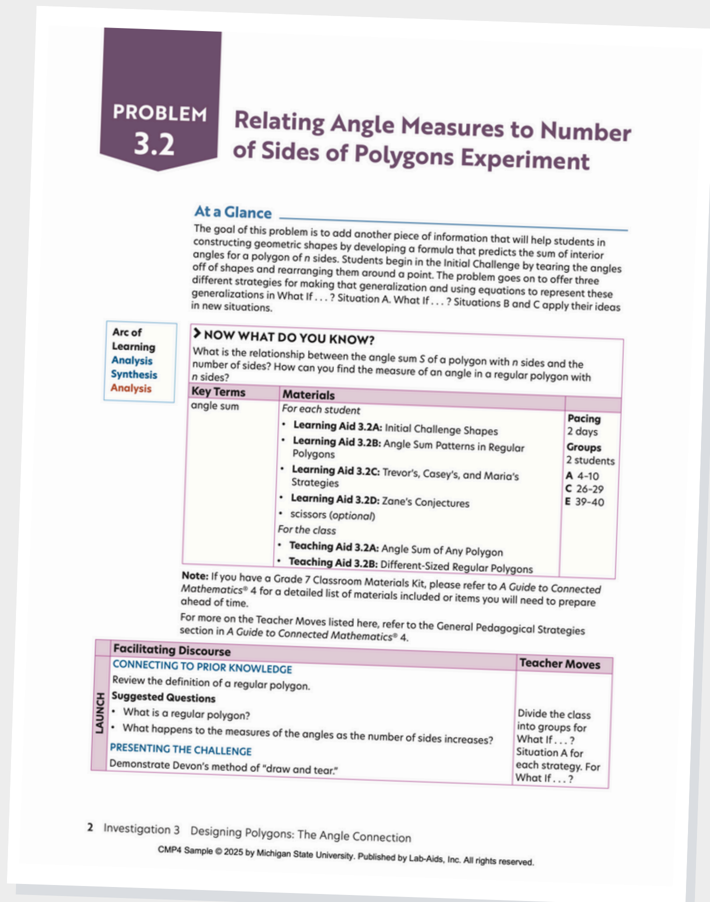
Grade 6 Teacher Edition

The Teacher Edition At a Glance provides Teacher Moves suggestions for Attending to Individual Learning Needs.



PROBLEM 3.2 **Relating Angle Measures to Number of Sides of Polygons Experiment**

At a Glance

The goal of this problem is to add another piece of information that will help students in constructing geometric shapes by developing a formula that predicts the sum of interior angles for a polygon of *n* sides. Students begin in the Initial Challenge by tearing the angles off of shapes and rearranging them around a point. The problem goes on to offer three different strategies for making that generalization and using equations to represent these generalizations in What If . . . ? Situation A. What If . . . ? Situations B and C apply their ideas in new situations.

Arc of Learning
Analysis
Synthesis
Analysis

> NOW WHAT DO YOU KNOW?

What is the relationship between the angle sum *S* of a polygon with *n* sides and the number of sides? How can you find the measure of an angle in a regular polygon with *n* sides?

Key Terms	Materials	
angle sum	*For each student* • **Learning Aid 3.2A:** Initial Challenge Shapes • **Learning Aid 3.2B:** Angle Sum Patterns in Regular Polygons • **Learning Aid 3.2C:** Trevor's, Casey's, and Maria's Strategies • **Learning Aid 3.2D:** Zane's Conjectures • scissors (*optional*) *For the class* • **Teaching Aid 3.2A:** Angle Sum of Any Polygon • **Teaching Aid 3.2B:** Different-Sized Regular Polygons	**Pacing** 2 days **Groups** 2 students **A** 4–10 **C** 26–29 **E** 39–40

Note: If you have a Grade 7 Classroom Materials Kit, please refer to *A Guide to Connected Mathematics®* 4 for a detailed list of materials included or items you will need to prepare ahead of time.

For more on the Teacher Moves listed here, refer to the General Pedagogical Strategies section in *A Guide to Connected Mathematics®* 4.

	Facilitating Discourse	Teacher Moves
LAUNCH	CONNECTING TO PRIOR KNOWLEDGE Review the definition of a regular polygon. **Suggested Questions** • What is a regular polygon? • What happens to the measures of the angles as the number of sides increases? PRESENTING THE CHALLENGE Demonstrate Devon's method of "draw and tear."	Divide the class into groups for What If . . . ? Situation A for each strategy. For What If . . . ?

2 Investigation 3 Designing Polygons: The Angle Connection

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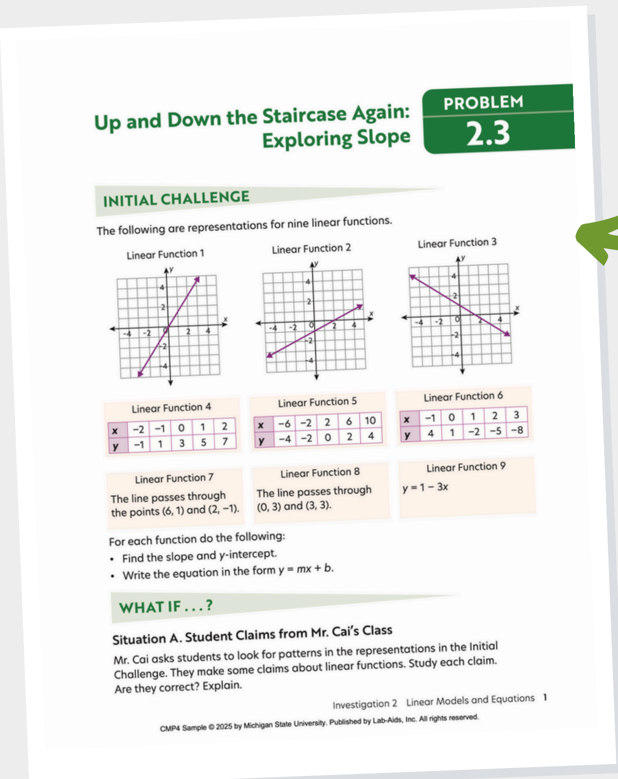
Grade 7 Teacher Edition

Equity
and Differentiation

Connected Mathematics

Connected Mathematics® 4 supports meaningful mathematics and language development in classrooms by using rich contextual mathematical tasks that invite students to share and compare strategies and approaches. Research-based strategies for supporting Multilingual Learners are embedded within the At a Glance and Extended Launch—Explore—Summarize sections of the Teacher Edition. Students can also listen to an audio version of the content in many different languages via the Lab-Aids digital portal.

Multilingual Learners



Up and Down the Staircase Again: Exploring Slope

PROBLEM 2.3

INITIAL CHALLENGE

The following are representations for nine linear functions.

Linear Function 1

Linear Function 2

Linear Function 3

Linear Function 4

x	−2	−1	0	1	2
y	−1	1	3	5	7

Linear Function 5

x	−6	−2	2	6	10
y	−4	−2	0	2	4

Linear Function 6

x	−1	0	1	2	3
y	4	1	−2	−5	−8

Linear Function 7

The line passes through the points (6, 1) and (2, −1).

Linear Function 8

The line passes through (0, 3) and (3, 3).

Linear Function 9

$y = 1 - 3x$

For each function do the following:

- Find the slope and y-intercept.
- Write the equation in the form $y = mx + b$.

WHAT IF . . . ?

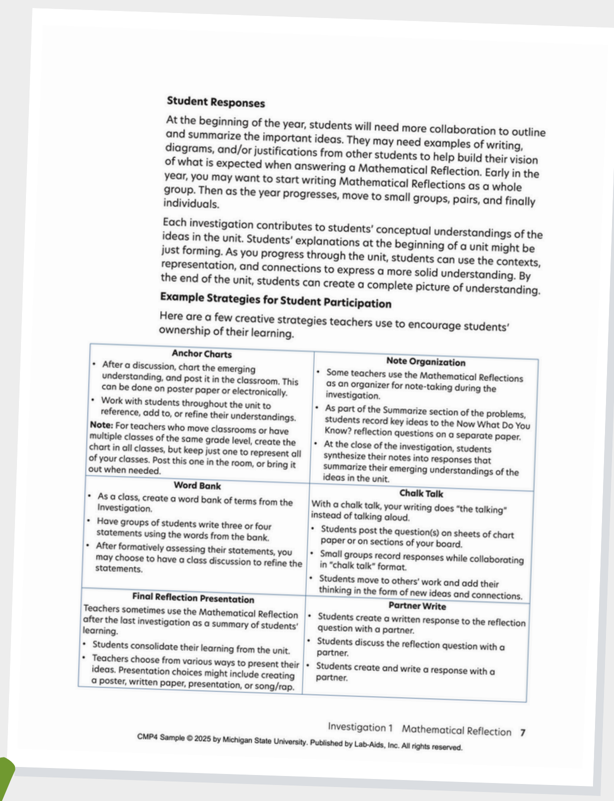
Situation A. Student Claims from Mr. Cai's Class

Mr. Cai asks students to look for patterns in the representations in the Initial Challenge. They make some claims about linear functions. Study each claim. Are they correct? Explain.

Investigation 2 Linear Models and Equations 1

Grade 8 Student Edition

Visual representations facilitate students' mathematical reasoning and problem solving, and provide all students access to engage in discourse about important mathematical ideas.



Student Responses

At the beginning of the year, students will need more collaboration to outline and summarize the important ideas. They may need examples of writing, diagrams, and/or justifications from other students to help build their vision of what is expected when answering a Mathematical Reflection. Early in the year, you may want to start writing Mathematical Reflections as a whole group. Then as the year progresses, move to small groups, pairs, and finally individuals.

Each investigation contributes to students' conceptual understandings of the ideas in the unit. Students' explanations at the beginning of a unit might be just forming. As you progress through the unit, students can use the contexts, representation, and connections to express a more solid understanding. By the end of the unit, students can create a complete picture of understanding.

Example Strategies for Student Participation

Here are a few creative strategies teachers use to encourage students' ownership of their learning.

Anchor Charts • After a discussion, chart the emerging understanding, and post it in the classroom. This can be done on poster paper or electronically. • Work with students throughout the unit to reference, add to, or refine their understandings. **Note:** For teachers who move classrooms or have multiple classes of the same grade level, create the chart in all classes, but keep just one to represent all of your classes. Post this one in the room, or bring it out when needed.	**Note Organization** • Some teachers use the Mathematical Reflections as an organizer for note-taking during the investigation. • As part of the Summarize section of the problems, students record key ideas to the Now What Do You Know? reflection questions on a separate paper. • At the close of the investigation, students synthesize their notes into responses that summarize their emerging understandings of the ideas in the unit.
Word Bank • As a class, create a word bank of terms from the Investigation. • Have groups of students write three or four statements using the words from the bank. • After formatively assessing their statements, you may choose to have a class discussion to refine the statements.	**Chalk Talk** With a chalk talk, your writing does "the talking" instead of talking aloud. • Students post the question(s) on sheets of chart paper or on sections of your board. • Small groups record responses while collaborating in "chalk talk" format. • Students move to others' work and add their thinking in the form of new ideas and connections.
Final Reflection Presentation Teachers sometimes use the Mathematical Reflection after the last investigation as a summary of students' learning. • Students consolidate their learning from the unit. • Teachers choose from various ways to present their ideas. Presentation choices might include creating a poster, written paper, presentation, or song/rap.	**Partner Write** • Students create a written response to the reflection question with a partner. • Students discuss the reflection question with a partner. • Students create and write a response with a partner.

Investigation 1 Mathematical Reflection 7

Grade 6 Teacher Edition

The Teacher Edition provides extensive support to guide teachers in creating opportunities for all learners to communicate their mathematical thinking and summarize their learning.

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Assessment

Variety to Meet Multiple Needs

Connected Mathematics®

Integrated assessments allow teachers to assess student progress and mastery of mathematics learning standards.

FORMATIVE ASSESSMENT FRAMEWORK

- Assessment features include a new formative assessment framework that emphasizes how teachers assess students during planning, teaching, and reflection of student learning.

MATHEMATICAL REFLECTIONS

- The Mathematical Reflections activity allows students to reinforce key concepts at a crucial learning stage. After each unit investigation, students revisit core questions, aiding in self-assessment and reflection. While this may feel unfamiliar at first, students gradually adapt to self-evaluation, guided by teacher feedback and peer work, enhancing their understanding of mathematics.

APPLICATIONS—CONNECTIONS—EXTENSIONS (ACE)

- The ACE provide additional practice to apply, connect, and extend students' mathematical concepts and skills. In the ACE, students are asked to compare, visualize, model, measure, count, reason, connect, and/or communicate their ideas in writing.

SUMMATIVE ASSESSMENTS

- **Checkup** Short, individual assessment instruments provide insight into student understanding of the baseline mathematical concepts and skills of the unit.
- **Partner Quiz** More complex than Checkup questions and more closely resemble the work the students do during class, which prepares them for the work done by STEM professionals. The Partner Quiz includes extensions of ideas students explored in class and provides insight into how students work together to apply the ideas from the unit to new situations. They are done with pairs of students and generally take more time.
- **Unit Test** Individual assessments that provide information on a student's ability to apply, refine, modify, and possibly extend the mathematical knowledge and skills acquired in the unit.

Materials

in Organized Classroom Kits

Connected Mathematics®

Classroom Material Kits for *Connected Mathematics*® 4 include everything a teacher needs for instruction. High-quality materials are organized by grade level with many items designed exclusively for the program.

Lab-Aids is known in science education for creating high-quality materials that are designed to last several years in a middle school classroom. Now we bring that expertise to the field of mathematics.

See the Classroom Materials Kits for all grade levels.

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